# Use of Multiple Antennas to Mitigate Carrier Phase Multipath in Reference Stations

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#### **BIOGRAPHY**

Jayanta Kumar Ray is a Ph.D. student in Geomatics Engineering at the University of Calgary. He received a B.E. and M.Tech. in Electronics Engineering from the Bangalore University and Indian Institute of Science, India, respectively. He has been involved in GPS research since 1992 in the area of GPS receiver hardware and software development, GPS with low cost sensors integration and multipath mitigation.

#### **ABSTRACT**

GPS carrier phase multipath is a major source of error in high precision static and kinematic differential positioning. It is especially prevalent in static applications whereby the multipath phase change is due only to satellite dynamics and can thus cause slowly varying bias errors. Though some of the currently available correlator-based techniques reduce high frequency multipath caused by distant objects, lower frequency multipath due to nearby objects still poses a problem. In this paper, an algorithm to estimate carrier phase multipath in a static environment using measurements from multiple closely-spaced antennas is developed and demonstrated. It is shown to significantly remove carrier phase multipath, especially the longer period effects.

In the proposed technique, the correlated nature of the multipath error across the antenna array is exploited along with the known geometry among the antennas. A Kalman filter was developed to estimate various parameters of the reflected signals using single difference (between antennas) carrier phase measurements available from the antenna array. The total multipath effect for every measurement at each antenna is then estimated and removed from the raw data. Carrier phase residual analysis performed before and after removing the multipath shows that up to a 60% improvement can be realized. A baseline test of the antenna assembly with respect to a second receiver shows that up to a 70%

improvement gain can be achieved in terms of position accuracy which demonstrates that the system is well suited for a reference station application.

# INTRODUCTION

In order to achieve position accuracies at the centimeter level, GPS carrier phase data must be used in differential mode. However, the presence of multipath has deleterious effects on carrier phase measurements, and limits the performance of GPS receivers used for surveying and other precision applications. Carrier phase multipath can theoretically reach as much as 5 cm and can be especially prevalent for reference receivers since they are installed at fixed locations. Multipath also plays a crucial role in precise attitude determination of space vehicles, as well as aircraft positioning during take-off, landing and taxiing (Braasch and van Graas, 1991).

Several methods have been devised to counter code range multipath, which in turn help reduce carrier phase multipath. Some of the antenna-related mitigation techniques include the use of a choke ring with an RF absorbing ground plane (Falkenberg et al., 1988; Lachapelle et al., 1989; Tranquilla and Karr, 1991), as well as antennas with special gain pattern characteristics (Counselman, 1998; Bartone and van Graas, 1998). However, these methods generally have the disadvantages of large size, weight and cost.

Improved receiver technologies were developed which have been found to be very effective in countering high frequency multipath. Some of these techniques are the Narrow Correlator<sup>TM</sup> spacing (Fenton et al., 1991; van Dierendonck et al., 1992), the Multipath Elimination Technique (MET<sup>TM</sup>) (Townsend and Fenton, 1994), MEDLL<sup>TM</sup> (van Nee, 1994), the Edge Correlator<sup>TM</sup> technique (Garin et al., 1996), the Strobe Correlator<sup>TM</sup> and the Enhanced Strobe Correlator<sup>TM</sup> (Garin and Rousseau, 1997). None of these techniques can effectively reduce low frequency multipath caused by nearby (< 30 m)

reflecting surfaces. Other techniques for multipath reduction that have been used involve antenna location strategies for optimal siting of the antenna, as well as long-term signal observation to infer multipath parameters facilitated by the changing reflection geometry (Weill, 1997).

Mitigation of carrier phase multipath has been addressed only recently due to the increase in applications that require higher accuracies. Axelrad et al. (1996) have used a signal-to-noise ratio (SNR) based technique to correct multipath errors in differential phase measurements in post-mission using a frequency resolution technique. Use of SNR measurements for multipath mitigation was further extended by Sleewaegen (1997). The geometrical aspects of reflection in combination with a special configuration of GPS antennas were exploited to detect and track multipath in a simulated multipath environment (Becker et al., 1994). Enhanced Strobe Correlator<sup>TM</sup> (Garin and Rousseau, 1997) or more recently Gated Correlator (McGraw and Braasch, 1999) reduces the correlator function width thereby eliminating high frequency carrier phase multipath. Moelker (1997) described various methods to mitigate multipath effects by using the Multiple Signal Classification (MUSIC) technique with multiple antennas and an extended Multipath Estimation Delay Lock Loop (MEDLL<sup>TM</sup>). Carrier phase multipath still remains a significant challenge, which has only been partially addressed to date by the above techniques.

Ray et al. (1998) used the spatial correlation of the multipath error between multiple closely-spaced antennas to remove multipath from the single difference carrier phase measurements across the antennas in the array. The objective of this paper is to extend this work and to demonstrate that it can be used to improve the position accuracy of a user receiver when multipath-corrected carrier phase measurements from the antenna array are used for differential positioning. Field data was simultaneously collected from the antenna array and a user receiver separated by 500 m to demonstrate the feasibility of this approach.

# MULTIPATH ERROR MODEL

A GPS receiver's incoming signal consists of direct and reflected components. The receiver cannot distinguish between them and therefore tracks the composite signal (Braasch, 1996). For continuous tracking of this composite signal, a local carrier is generated by a numerically controlled oscillator that tries to keep up with the incoming composite carrier frequency and phase. The phase difference between the composite signal phase and the direct signal phase is the multipath phase error.

The effect of multipath on the carrier phase is described by several researchers, e.g. Braasch (1996); van Nee (1995); Ray et al. (1998); and Ray & Cannon (1999). From Ray et al. (1998) the error is given by

$$\Delta \Psi = \arctan\left(\frac{R(\tau - \delta)\alpha\sin\gamma}{R(\tau) + R(\tau - \delta)\alpha\cos\gamma}\right)$$
(1)

where,

 $\Delta \Psi$  is the multipath error (rad)

R is the auto correlation function

τ is the delay of the direct signal with respect to locally generated code (s)

 $\delta$  is the delay of the multipath signal with respect to the direct signal (s)

α is the reflection coefficient, and

γ is the reflected signal phase (rad).

This may be graphically represented as shown in Figure 1.

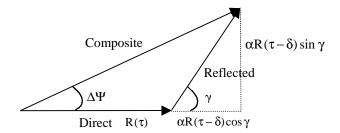


Figure 1: Direct, reflected and composite signal and multipath error.

Equation 1 can be normalized by dividing the numerator and denominator by  $R(\tau)$  and describing  $\alpha R(\tau-\delta)/R(\tau)$  as  $\alpha_0$  to give

$$\Delta \Psi = \arctan\left(\frac{\alpha_0 \sin \gamma}{1 + \alpha_0 \cos \gamma}\right). \tag{2}$$

From equations (1) and (2) it can be observed that the multipath parameters are the reflection coefficient, the reflected signal phase, and the path delay. By combining the reflection coefficient and the path delay, a modified reflection coefficient ( $\alpha_0$ ) was introduced. This reduces the number of multipath parameters to two.

Multipath error is spatially correlated as was shown in Ray et al. (1998). Therefore, if several antennas are placed close-by, the multipath experienced by each antenna will be similar (provided that the reflector is large in size). The difference in the carrier phase error at two closely-spaced antennas is then given by

where,

 $\Delta \Psi_{0i}$  is the difference in multipath error for antennas 0 and i (rad)

 $\Delta \Psi_0$  is the multipath error at antenna 0 (rad)

 $\Delta \Psi_i$  is the multipath error at antenna i (rad)

 $\alpha_0$  is the modified reflection coefficient for both antennas

 $\gamma_0$  is the reflected signal phase in antenna 0 (rad), and

 $\gamma_i$  is the reflected signal phase in antenna i (rad).

If the antennas point in the same direction in the multiantenna system, the effect of phase center variation will be removed. Pointing of the antennas also helps in ensuring an identical gain pattern for each antenna (in any direction) and will introduce identical amplification or attenuation to the incoming signals from a particular direction. In such a situation, the modified reflection coefficient for each antenna in a closely-spaced multiantenna system can be assumed to be the same.

The same is not true for a reflected signal phase, which is highly sensitive to the reflected signal path delay. For a path delay of 10 cm, the reflected signal phase will have an 180 degree phase shift for an L1 signal carrier. In a multi-antenna assembly, it is possible to relate the reflected signal phase in each antenna in terms of the reflected signal direction and inter-antenna geometry. This is given by

$$\gamma_i = \gamma_0 + \frac{2\pi}{\lambda} a_{0i} \cos(\varphi_0 - \phi_{0i}) \cos \theta_0 \tag{4}$$

where,

 $\lambda$  is the carrier wavelength (m)

 $a_{0i}$  is the distance between antennas 0 and i (m)

 $\phi_{0i}$  is the azimuth of the vector from antenna 0 to i (rad)

 $\theta_0$  is the elevation of the reflected signal (rad) and

 $\phi_0$  is the azimuth of the reflected signal (rad).

Equation (4) forms the basis of using carrier phase measurements from several antennas to estimate a common set of multipath parameters.

# MULTIPATH MITIGATION TECHNIQUE

To effectively estimate multipath errors in carrier phase measurements, all the multipath signals to the antenna need to be considered. One way to do this is to determine all the dominant frequency components of multipath from the SNR curve (corrected for satellite elevation and antenna gain pattern) from which the multipath errors can be reconstructed (Axelrad et al., 1994). Another way is to estimate the multipath parameters of all the multipath signals and add their effects algebraically. In the current approach, composite multipath parameters due to

reflections from all sources in the environment are estimated from which the total effects of multipath can be computed.

An Extended Kalman Filter (Brown and Hwang, 1992; Gelb, 1979; Maybeck, 1994) was developed to estimate the composite multipath parameters. The state vector for the Kalman filter is,

$$\mathbf{x} = [\alpha_0 \ \gamma_0 \ \theta_0 \ \phi_0]^{\mathrm{T}} \tag{5}$$

The first two elements are the composite multipath parameters and were defined with regards to equation (3). The other two elements describe the direction of the reflected signal and were defined in equation (4)

Single difference (between antenna) carrier phase measurements are used to update the state variables since they are free from atmospheric delay errors, satellite orbital errors, and satellite clock errors. The measurement is given by the expression (Lachapelle, 1997)

$$\Delta\Phi_{0i} = \Delta\rho_{0i} + \Delta N_{0i}\lambda + c\Delta t_{0i} + \Delta\epsilon_{\phi 0i} + \Delta\epsilon_{MP0i}$$
 (6)

where,

ΔΦ is the measured carrier phase single difference between antennas 0 and i (m)

Δρ is the range difference due to spatial separation between antennas (m)

 $\Delta N$  is the integer ambiguity difference

 $c\Delta t$  is the receiver clock bias difference (m)

 $\Delta \varepsilon_{\omega}$  is the carrier phase noise difference (m), and

 $\Delta \varepsilon_{MP}$  is the carrier phase multipath error difference (m).

Each antenna in the multi-antenna system is connected to a different receiver. If the receivers are driven by a common clock and their carrier phase measurements are corrected for the antennas' spatial separations, then the single difference carrier phase measurements contain only the difference of the integer ambiguity, multipath and phase noise. As the multipath and phase noise together are much smaller compared to the carrier wavelength, the integer cycles can be easily removed and the resultant modified single difference contains only the difference of multipath and phase noise between the antennas. Neglecting phase noise this gives.

$$\Delta \Psi_{0i} = \Delta \Phi_{0i} . \tag{7}$$

Therefore the measurement vector is given by

$$\mathbf{z} = [\Delta \Psi_{01} \ \Delta \Psi_{02} \ \dots \dots \Delta \Psi_{0m-1}]^{\mathrm{T}}$$
 (8)

where m is the number of antennas used.

The design matrix for the filter is calculated by computing the partial derivative of equation (3) with respect to each parameter and using equation (4). This gives the following form,

$$\mathbf{H} = \begin{bmatrix} \frac{\delta(\Psi_{01})}{\delta\alpha_{0}} & \frac{\delta(\Delta\Psi_{01})}{\delta\gamma_{0}} & \frac{\delta(\Delta\Psi_{01})}{\delta\theta_{0}} & \frac{\delta(\Delta\Psi_{01})}{\delta\phi_{0}} \\ \frac{\delta(\Delta\Psi_{02})}{\delta\alpha_{0}} & \frac{\delta(\Delta\Psi_{02})}{\delta\gamma_{0}} & \frac{\delta(\Delta\Psi_{02})}{\delta\theta_{0}} & \frac{\delta(\Delta\Psi_{02})}{\delta\phi_{0}} \\ ---- & ---- & ---- \\ \frac{\delta(\Delta\Psi_{0m-1})}{\delta\alpha_{0}} & \frac{\delta(\Delta\Psi_{0m-1})}{\delta\gamma_{0}} & \frac{\delta(\Delta\Psi_{0m-1})}{\delta\theta_{0}} & \frac{\delta(\Delta\Psi_{0m-1})}{\delta\phi_{0}} \end{bmatrix}$$
(9)

The modified single difference residual carrier phase measurement for a particular satellite is input to the multipath mitigation filter to adaptively estimate the parameters of the composite multipath due to all reflectors affecting the carrier phase. After the parameters are estimated by the filter, it is possible to determine the multipath errors in the carrier phase at each antenna by using equations (2) and (4).

To evaluate the performance of this technique in a realistic scenario double difference residuals were calculated over a baseline with the antenna array acting as the reference station, and second receiver acting as a 'user' receiver. This approach is described below.

The double difference carrier phase measurement between an antenna in a multi-antenna system and a user antenna is given by (assuming a baseline short enough that atmospheric and orbital errors are negligible)

$$\Delta \nabla \Phi^{jk}_{0i} \!=\! \Delta \nabla \rho^{jk}_{0i} \!+\! \Delta \nabla N^{jk}_{0i} \lambda \!+\! \Delta \nabla \epsilon^{jk}_{\phi 0i} \!+\! \Delta \nabla \epsilon^{jk}_{MP0i} \eqno(10)$$

where,

 $\Delta \nabla$  is the double difference operator

i is the user antenna index

i is the base satellite index, and

k is the non-base satellite index.

For experimental purposes, if the positions of the antennas in the multi-antenna assembly and the user antenna are known, the phase difference due to spatial separation can be easily removed. Also, in this case the double difference integer ambiguities were known and were removed. The resultant residual double difference carrier phase measurements therefore contain the double difference of the multipath errors and phase noise.

Assuming that the user antenna site has negligible multipath, the residual double difference carrier phase measurements contain the single difference multipath between the base satellite and another satellite in the

antenna assembly. This is deemed the *measured* multipath error. The single difference of the *estimated* multipath between the base satellite and another satellite may be computed using the proposed multipath mitigation technique. Statistics computed for the measured multipath error versus the estimated error can then be compared to evaluate the performance of this technique.

The technique described above should remove multipath from the reference stations' measurements and generate corrections for the user, which should in turn improve the position of the user. Therefore, it is important to analyze the user position accuracy when the reference station data is corrected.

Baseline testing was performed to analyze the user position accuracy. The estimated multipath for each satellite in each antenna in the antenna array is removed from the measurements. The position of the user is then computed in differential mode, first with the uncorrected carrier phase data from the reference station, and secondly using the multipath-corrected measurements. A comparison between the position errors with and without corrections indicates the achievable improvement in user position using such a system.

# TEST DESCRIPTION

The proposed multipath mitigation technique was tested using the <u>Multipath Simulation and Mitigation (MultiSim)</u> software (Ray and Cannon, 1999). After having successfully demonstrated the mitigation of multipath using the above model on simulated multipath, the same approach was applied to field data as discussed below.

A special antenna array was assembled whereby a thick aluminum plate was used to rigidly mount six antennas close together. NovAtel Model 521 antennas were used, as they are small with a diameter of approximately 5.6 cm.



Figure 2: Antenna array assembly.

NovAtel BeeLine™ receivers were used for data collection (Ford et al., 1997), wherein all receivers were driven by the same external rubidium oscillator. The antenna assembly was placed on a surveyed pillar (on the roof of the Engineering Building) where there are concrete sidewalls of approximately 3 m in height on the

east side and 1 meter in height on the south side which are expected to cause low frequency multipath errors.

The antenna array was used as a reference receiver for the tests. A NovAtel Millennium receiver with a choke ring antenna (user) was placed in an open field where there were no major objects in the range of 80 to 100 m from the antenna. The baseline separation from the antenna array to the MilLennium was approximately 500 m. Only L1 data from the MilLennium was used.

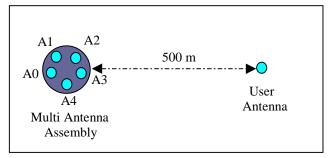


Figure 3: Experimental setup

A series of tests were conducted in October, 1998 whereby raw L1 carrier phase data was collected at 1 Hz rate for about two to three hours per test using the antenna array and the user receiver setup described above. The carrier-to-noise ratio ( $C/N_0$ ) of the multi-antenna system data was first analyzed for inter-antenna coupling using data collected on October 7. This experiment was repeated using data collected on October 8. A second analysis was performed on the double difference residuals between the antenna array and the user receiver with and without multipath correction using data collected on October 20. A final analysis was done in the position domain using the same data set to assess the impact of the mitigation technique on the achievable accuracy of the user receiver.

Positions of the antennas in the multi antenna assembly were accurately surveyed to determine the inter-antenna vectors, and hence the relative geometry. The University of Calgary's Semikin<sup>TM</sup> software (Cannon, 1993) was used for this purpose.

In each observed satellite single difference measurements between the antennas in the array were formed and corrected for their spatial separation. Integer ambiguities were also removed. The residual single difference measurements were used as input to the multipath mitigation filter. The Kalman filter estimated the unknown parameters of the composite multipath signal. By using equations (2) and (4), the multipath error at each antenna was computed.

The single difference multipath error between the base satellite and another satellite in each antenna in the assembly was then computed and deemed the estimated multipath error. The University of Calgary's Flykin<sup>TM</sup> software (Lu et al., 1994) was used to determine the double difference carrier phase residuals between antennas in the assembly and the user receiver to compute the measured multipath.

#### TEST RESULTS

#### **Antenna Coupling Test**

Figure 4 shows the  $C/N_0$  of the center antenna in the antenna assembly versus the elevation of all the satellites visible during the October 7 test period. A wide spread in the signal power is observed even up to  $60^{\circ}$  elevation angles. In Figure 5,  $C/N_0$  values are plotted for the same antenna with respect to the azimuth of all available satellites. Different shades are used to indicate various ranges of elevation angles. The signal power has a large dip or non-uniformity around the  $180^{\circ}$  azimuth angle mark. This is likely to be due to heavy coupling of the center antenna by the peripheral antennas.

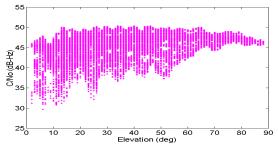


Figure 4: Signal power to noise power spectral density of the center antenna in the six-antenna assembly with respect to the elevation of the satellites.

 $C/N_0$  in a receiver depends upon several factors including the nominal received power, elevation angle of the satellite, foliage attenuation, line loss, antenna gain pattern and multipath (Spilker, 1996). The specified received signal level peaks at  $40^\circ$  elevation by approximately 2 dB with respect to the nominal signal level at the low elevation angle to account for the extra path loss (Spilker, 1996). In an ideal static environment, the  $C/N_0$  values will have a parabolic signature due to the line of sight traversing the antenna gain pattern (Axelrad, 1996). If  $C/N_0$  values for all visible satellites are plotted together, they should form a thin band.

Due to the large variation in the  $C/N_0$  values a second test was conducted with the center antenna removed. Figures 6 and 7 show the  $C/N_0$  values of all satellites for one of the peripheral antennas in the antenna assembly. In this case the signal power spread is narrow compared to the previous case.

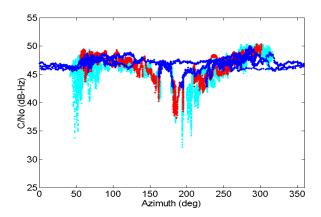


Figure 5: Signal power to noise power spectral density of the center antenna in the six-antenna assembly with respect to the azimuth of the satellites. (Light shade:  $20^{\circ} \le Elv < 40^{\circ}$ ; Medium shade:  $40^{\circ} \le Elv < 60^{\circ}$ ; Dark shade:  $60^{\circ} \le Elv < 90^{\circ}$ ).

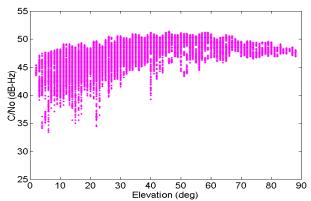


Figure 6: Signal power to noise power spectral density of a peripheral antenna in the five-antenna assembly with respect to the elevation of satellites.

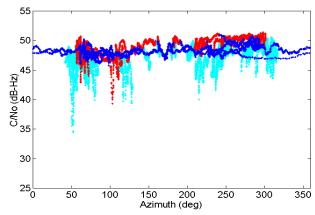


Figure 7: Signal power to noise power spectral density of a peripheral antenna in the five-antenna assembly with respect to the azimuth of the satellites. (Light shade:  $20^{\circ} \le \text{Elv} < 40^{\circ}$ ; Medium shade:  $40^{\circ} \le \text{Elv} < 60^{\circ}$ ; Dark shade:  $60^{\circ} \le \text{Elv} < 90^{\circ}$ ).

This experiment was repeated again using different data sets and for other peripheral antennas and similar results were found. This suggests that the center antenna in the six-antenna assembly is highly coupled and its gain pattern is disturbed substantially. Therefore, the center antenna was removed from the antenna assembly and all the subsequent experiments were carried out with only five antennas.

#### **Carrier Phase Residual Test**

In order to demonstrate the effectiveness of the multipath mitigation technique at the multi-antenna array, data from October 20, 1998 was used for analysis. Single difference phase residuals for each of the antenna pairs were derived for satellite 31. This is a low elevation (23°-35°) satellite and is likely to have be more affected by multipath. The phase residuals are shown in Figure 8 and contain low to medium frequency oscillations with variable amplitude due to all the reflectors in the environment. It also contains high frequency phase noise.

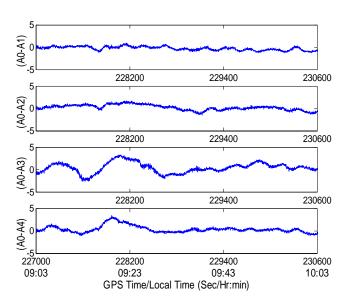


Figure 8: Single difference residual carrier phase error for SV 31 on October 20, 1998 (Y-axis units in cm; A0-An denotes single difference between antennas 0 and n).

Figure 9 shows the estimated parameters of the composite reflected signal for this satellite as determined from the filter. As expected, the parameters vary with time to track all the reflections from the environment. The reflected signal phase changes, which causes positive and negative multipath errors. The reflected signal phase at the other antennas were computed using equation (4). The multipath error was then computed by using the estimated parameters in equation (2). Estimated parameters for other satellites had similar characteristics.

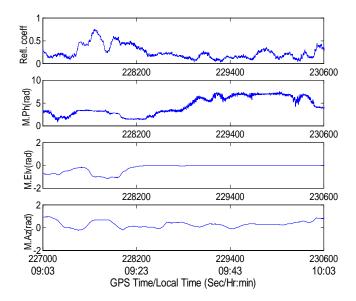


Figure 9: Estimated composite reflected signal parameters for SV 31 on October 20.

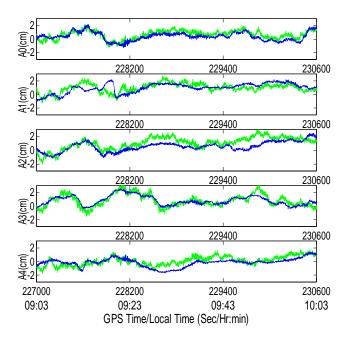


Figure 10: Measured multipath (light shade) and estimated multipath (dark shade) for SV 31. (A0...An denote Antenna 0....n)

Satellite 23, being the highest elevation (86°-60°) satellite, was used as the base satellite for double differenceing. Multipath was estimated for this satellite at each antenna in the array using the same procedure as satellite 31. The differences in multipath errors between satellites 23 and 31 in each antenna were computed and are shown in Figure 10 using a dark shaded line (estimated multipath). Also, in the same figure double difference carrier phase residuals between the user and

each antenna in the array are shown using a light shaded line (*measured multipath*). If the multipath in the user antenna were zero, then the estimated and measured multipath signals would coincide. In the present case, the user antenna was in an open environment and should therefore have limited multipath. From the figure it can be seen that the estimated multipath closely follows the measured multipath.

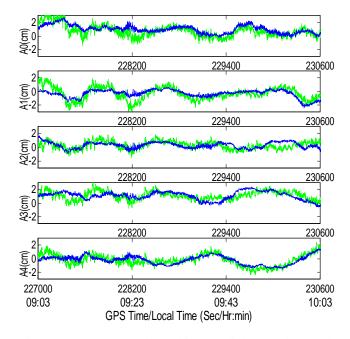


Figure 11: Measured multipath (light shade) and estimated multipath (dark shade) for SV 9. (A0...An denote Antenna 0...n).

Multipath was estimated in other low elevation satellites and compared with their measured values. Figure 11 shows the estimated and measured multipath for satellite 9. In this case the estimated values also closely follow the measured values.

Multipath for satellites 1, 9, 17, 23 and 31 in each antenna in the assembly were estimated and removed from the raw data. The double difference carrier phase measurements between the user and the antennas in the assembly were recomputed using the corrected measurements. Figure 12 shows the residuals before and after correction. As can be seen, corrected residuals have much lower oscillations.

Table 1 gives the residual statistics before and after multipath correction for the low elevation satellites. A carrier phase noise value of 0.15 cm was assumed which contributes a 0.3 cm RMS error to the double difference residuals. This was removed from the residuals in order to isolate the error due to multipath. There were small mean values in the measured multipath, which may have occurred due to a receiver line bias or an insufficient

accuracy estimate in the inter-antenna geometry computation in the antenna array. These were removed from the measured multipath, as carrier phase multipath has zero mean (Ray and Cannon, 1999).

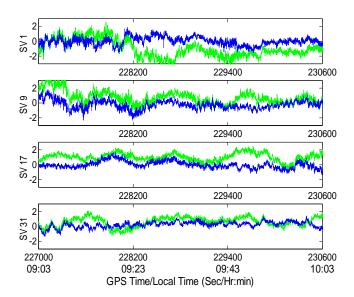


Figure 12: Double difference carrier phase residual before (light shade) and after (dark shade) correction for Antenna 0. (Y-axis in cm).

It can be observed that this method is very effective in a high multipath environment by decreasing the magnitude of the residuals up to 60%. An average of 20% improvement were observed for all satellites in all antennas. However, when the magnitude of multipath is low, or there is a high frequency component, this method is not as effective. In some cases, it deteriorates slightly (e.g. antenna 2 and 3 for satellite 1 in the table) although the absolute values of the residuals are still quite small (around 0.6 cm). It should be noted that in the table the user antenna is assumed to have no multipath, which is not a reality. Therefore the actual improvement using this technique is conservatively reported in the table.

# **Position Accuracy Test**

A baseline test was carried out to analyze the impact in the user position accuracy when using corrected carrier phase measurements from the multi-antenna system. The position of the user was first computed using uncorrected carrier phase measurements from the reference station. Antenna A0 was selected from the multi-antenna system to be the reference receiver. Ambiguities were fixed to their integer values and positions were generated at 1 Hz. A few satellites which were not available for the entire duration of the experiment were not used in this analysis. The position error time series is shown in Figure 13 using a shaded light line. Positions were then recomputed using corrected measurements and the corresponding errors are

shown in the same figure using a dark line. It can be observed that the position errors using corrected measurements have smaller magnitudes.

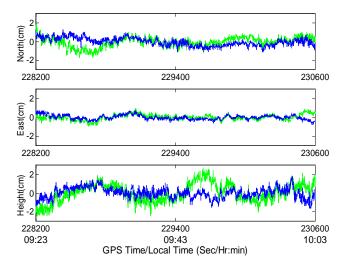


Figure 13: Position error in Antenna 0 before (light shade) and after (dark shade) multipath correction of carrier phase measurements.

This analysis was repeated using each of the antennas in the multi-antenna system as the reference, and the statistics are shown in Table 2. The 3D position error was computed using the following equation,

$$\sigma_{\mathrm{3D\,error}} = \sqrt{\sigma_{northing\,error}^2 + \sigma_{easting\,error}^2 + \sigma_{height\,error}^2}$$

An average PDOP of 2.36 was observed during the test period. As the RMS of position error is equal to the PDOP multiplied by the RMS of measurement error, a phase noise of 0.3 cm (RMS) would cause 0.71 cm  $(\sigma_{\text{nom}})$  of position error (RMS). This was removed from the total position error to isolate the error caused by multipath.

Up to a 70% improvement in the 3D position was achieved using this technique. Antenna 2 shows a negative improvement as the residuals slightly deteriorated in this case (although the absolute error is quite small)

Overall, a significant improvement in position accuracy is observed which verifies the effectiveness of the technique. From Tables 1 and 2 it can be seen that the higher the multipath in the antenna, the better the multipath reduction and the higher the position accuracy improvement.

These results have demonstrated that significant improvement in user position accuracy can be achieved by using such a multi-antenna system as a reference station.

Table 1: Statistics of double difference carrier phase measurement residuals with and without multipath correction

			Multipath +	Multipath +	Multipath	Multipath	Multipath
			Noise σ	Noise σ	σ	σ	Reduction
SV	Elevation	Antenna	before	after	before	after	([3]-[4])/[3]
No.	Angle	No.	correction	correction	correction	correction	σ
			(cm)	(cm)	(cm)	(cm)	(%)
			[1]	[2]	[3]	[4]	
SV 1	14° - 42°	A0	1.14	0.55	1.09	0.46	58.00
		A1	0.80	0.53	0.74	0.44	40.55
		A2	0.62	0.69	0.55	0.62	-14.00
		A3	0.65	0.65	0.58	0.58	-0.74
		A4	0.70	0.70	0.63	0.64	-0.16
	13° - 27°	A0	0.86	0.58	0.80	0.49	38.91
SV 9		A1	0.93	0.71	0.88	0.64	27.40
		A2	0.54	0.56	0.45	0.48	-5.12
		A3	0.61	0.65	0.53	0.57	-7.16
		A4	0.81	0.54	0.75	0.45	40.11
SV 17	48° - 21°	A0	0.53	0.47	0.44	0.37	16.36
		A1	0.86	0.53	0.80	0.43	45.89
		A2	0.47	0.52	0.36	0.43	-19.44
		A3	0.53	0.54	0.44	0.45	-3.08
		A4	0.60	0.61	0.52	0.53	-1.60
SV 31	23° - 35°	A0	0.62	0.40	0.55	0.26	52.54
		A1	0.65	0.65	0.58	0.57	1.20
		A2	0.89	0.62	0.84	0.55	34.75
		A3	0.89	0.44	0.84	0.32	61.90
		A4	0.65	0.56	0.58	0.48	17.02

Table 2: Statistics of user position accuracy with and without multipath correction

Antenna No.	Northing error   o  (cm)  [1]		Easting  o  (cm [2]	n)	Height error  σ (cm) [3]		3D position error due to multipath + noise $\sigma$ (cm) $\sqrt{[1]^2 + [2]^2 + [3]^2}$ [4]		3D position error due to multipath $\sigma$ (cm) $\sqrt{[4]^2 - \sigma_{nom}^2}$ [5]		Improvement in 3D position accuracy by multipath correction (5[a]-5[b])/ 5[a]
	Before	After	Before	After	Before	After	Before	After	Before	After	
	[a]	[b]	[a]	[b]	[a]	[b]	[a]	[b]	[a]	[b]	
A0	0.60	0.45	0.29	0.25	1.00	0.57	1.20	0.77	0.97	0.30	69.34
A1	0.54	0.40	0.31	0.42	1.13	0.61	1.29	0.84	1.08	0.46	57.79
A2	0.43	0.53	0.40	0.38	0.82	0.81	1.01	1.08	0.72	0.87	-6.93
A3	0.47	0.49	0.34	0.29	1.04	0.87	1.19	1.04	0.96	0.76	20.47
A4	0.39	0.48	0.31	0.25	1.31	0.69	1.40	0.88	1.21	0.52	57.22

 $<sup>{}^*\</sup>sigma_{\text{nom}}$  is the nominal position error due to phase noise = PDOP\*standard deviation of phase noise.

#### **CONCLUSIONS**

A technique has been developed and described to reduce the effect of multipath on carrier phase measurements using a multi antenna system. Tests were carried out at the University of Calgary using a multi-antenna assembly consisting of five closely-spaced antennas. NovAtel BeeLine<sup>TM</sup> receivers driven by a common oscillator were used for the experiment.

The technique exploits the spatial correlation of multipath along with the known geometry among the antennas to estimate the multipath parameters. The multipath error for each satellite at each antenna was then estimated and removed from the raw measurement. Residuals with and without multipath correction were obtained and a comparison of residual statistics showed that the technique is very effective in a high multipath environment reducing its effect up to 60% in the user receiver. A baseline vector between the antennas in the assembly and the user antenna was computed with and without multipath-corrected measurements and up to a 70% improvement in accuracy for the user position was achieved.

The technique shows promise to reduce carrier phase multipath, especially the low frequency multipath which can not be reduced using the existing correlator-based techniques. This technique has potential to be used in real time. SNR information may further be included along with the phase measurements for more accurate estimation of the multipath parameters. A higher number of antennas may also be used in the assembly for added robustness of the system.

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