Attitude Determination Using Multipath Mitigation on Multiple Closely-Spaced Antennas

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ABSTRACT

The effect of multipath mitigation on accurate attitude determination using multiple antennas, spaced 10 to 14 cm apart, is investigated The correlated nature of multipath, along with the known geometry among the antennas, are used to aid in the extraction of the direct carrier phase from the multipath-corrupted carrier phase measurement. The mathematical model of the multipath effects on carrier phase measurements and the Kalman filter implemented to estimate these errors are described. It is then tested on static field data collected on a roof whereby the model adaptively estimates the parameters of the composite multipath signal. Initial results demonstrate substantial mitigation of carrier phase multipath signals from multiple sources using this technique under static conditions. The RMS value of the average multipath error on a single differenced carrier phase measurement data is typically reduced by over 70% percent. This technique estimates the parameters of the composite multipath signal and removes the error due to all multipath signals. Attitude determination is performed using the raw carrier phase measurements and the multipath mitigated measurements and the accuracy of the attitude angles from the two cases are compared and show an improvement in the 30-70% range.

INTRODUCTION

The ability of GPS to provide accurate attitude components has been demonstrated using several platforms and a variety of operational conditions (see van Graas and Braasch, 1991; Cohen and Parkinson, 1992; Cohen et al., 1993; Cannon and Sun, 1996). One of the limiting factors for improved performance is carrier phase multipath, which has a maximum magnitude of about 5 cm on L1, and which does not cancel during differential processing (Reichert and Axelrad, 1999). Attitude accuracies have been demonstrated at the level of a few arcmin to several degrees, depending on the antenna separation, but new techniques will be required for multipath mitigation in order to further improve these values.

Some work has been done on multipath reduction and its application to attitude determination (e.g. Axelrad et al., 1994). In this paper, a technique is presented for carrier phase multipath mitigation which is based on the estimation of multipath parameters using relative carrier phase measurements from an array of closely-spaced antennas. This technique has been presented previously (Ray et al., 2000; Ray, 2000) in the context of kinematic positioning, and in this paper it is extended to attitude determination.

In this paper GPS attitude determination is described briefly, along with the multipath mitigation method developed. Field test results are presented with and without multipath mitigation and conclusions regarding its effectiveness in attitude determination applications are discussed.

GPS ATTITUDE DETERMINATION

There are commercial GPS-based attitude determination systems that use receivers dedicated for this purpose (Ferguson et al, 1991). An alternative to the above approach is to develop a non-dedicated attitude determination system comprised of three or more independent GPS receivers mounted on the platform. One advantage of such a system is flexibility since the receivers can be used for a variety of applications in addition to attitude determination, e.g. Sun (1994). Cost-effectiveness may also be gained through the utilization of low-cost GPS receivers which output the carrier phase observable. A series of receiver pairs, designed for heading and pitch determination, were used in the following analysis of a full 3-D attitude system. Therefore, the system overall can be considered non-dedicated system since the receiver pairs operate independently.

The GPS data was processed using the University of Calgary's MULTINAV™ software which estimates roll, pitch and heading using carrier phase measurements from three or more antennas (Sun, 1994).

The body frame, which is needed for definition of the platform attitude, was realized by three antennas, as shown on Figure 1 below. The body frame can be measured directly using a theodolite or can be determined by GPS initialization, which is typically more convenient.

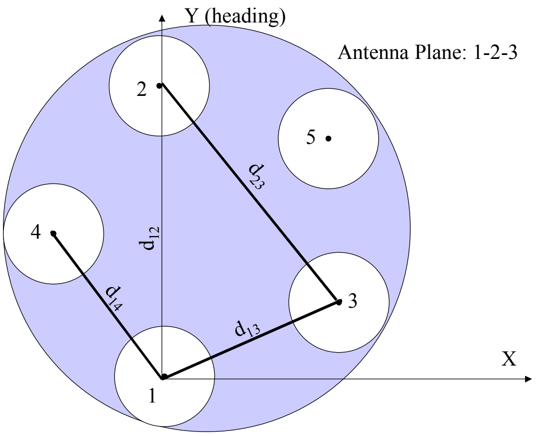

Figure 1: Body frame defined by GPS antennas.

Attitude components, i.e. roll, pitch and heading, are estimated via a least squares approach using the inter-antenna vectors as quasi-observables. Suppose $r_i^b = (x_i^b, y_i^b, z_i^b)^T$ are the body-frame coordinates of the i-th antenna which were previously estimated. The measurements are $r_i^n = (x_i^n, y_i^n, z_i^n)^T$, the local level coordinate of the i-th antenna, which are determined from the differential GPS carrier phase solution. These coordinates satisfy the following equation

$$\begin{vmatrix} r_2^b \\ r_3^b \\ r_4^b \end{vmatrix}^T = R_n^b(\varphi,\theta,\psi) \begin{pmatrix} r_2^n \\ r_3^n \\ r_4^n \end{pmatrix}^T \qquad (1a)$$

where φ, θ, and ψ, are roll, pitch and heading, respectively, and

$$R_n^b(\varphi,\theta,\psi) = \qquad (1b)$$

$$\begin{pmatrix} c(\psi)c(\varphi)-s(\psi)s(\theta)s(\varphi) & s(\psi)c(\varphi)+c(\psi)s(\theta)s(\varphi) & -c(\theta)s(\varphi) \\ -s(\psi)c(\theta) & c(\psi)c(\theta) & s(\theta) \\ c(\psi)s(\varphi)+s(\psi)s(\theta)c(\varphi) & s(\psi)s(\varphi)-c(\psi)s(\theta)c(\varphi) & c(\theta)c(\varphi) \end{pmatrix}$$

where c() is a cosine function and s() is a sine function. When there are three antennas on the platform, a unique solution is generated, whereas additional antennas provide redundancy. These equations can be solved using a least squares adjustment by minimizing the cost function

$$J(\varphi,\theta,\psi) = \left\| (r^b - R(\varphi,\theta,\psi) r^n \right\|^2 . \qquad (2)$$

The least squares method has many advantages over other methods such as a direction computation of attitude (Lu et al., 1993). It can accommodate more antennas and attitude is less affected by multipath from a single antenna since it is based on a least squares fit of all antenna positions.

Further details on the methodology used in the attitude determination algorithms are given in Lachapelle et al. (1994) and Lu (1994).

CARRIER PHASE MULTIPATH MITIGATION USING MULTIPLE ANTENNAS

The multipath mitigation algorithm processes raw carrier phase measurement data from an antenna array before the data is used for attitude determination (or kinematic positioning if the array is used as a reference station). Although this technique can be used to correct phase measurements in real-time, at present it is implemented in post mission.

In the mitigation algorithm, a virtual reflector represents the total sum of all the associated reflectors and is modeled as a single reflector. For this reason it is appropriate to analyze the concept of multipath mitigation using the single reflector case.

The reflection of a satellite signal can be viewed from a geometrical perspective. For example, if the satellite is far away, the GPS signal can be assumed to arrive as parallel rays at two closely-spaced antennas. A plane wavefront, perpendicular to the line of sight, can be assumed to have the same carrier phase. After reflection from a plane reflector, the parallel incident signals remain parallel and thus phase propagation takes place through the advancement of the plane wavefront. Therefore, the phase of the reflected signal at each antenna phase center in a group of closely-spaced antennas is a function of the reflected signal direction (i.e. azimuth and elevation) as well as the relative geometry of the antennas with respect to each other.

In Figure 2, two antennas are placed at Antennas 0 and 1. Each of the antennas receives a direct signal from the satellite and a reflected signal from a nearby plane object. A wavefront perpendicular to the indirect signal at Antenna 0 will have the same phase for all the other parallel reflected rays from the same object. Therefore the phase of the signal at Antenna 1 is given by,

$$\gamma_1 = \gamma_0 + \frac{2\pi}{\lambda} a_{01} \cos(\varphi_0 - \phi_{01}) \cos\theta_0 \qquad (3)$$

where,

- γ_0 is the phase of the signal at Antenna 0
- a_{01} is the distance between 0 and 1
- φ_0 is the azimuth of the reflected signal
- ϕ_{01} is the azimuth of the vector 0-1, and
- θ_0 is the elevation of the reflected signal.

If the phase and direction of the reflected signal at Antenna 0 are known, the phase at Antenna 1 can be computed from the known geometry between the two antennas. This relationship is exploited to estimate the reflected signal phase at each of the antennas in the array thereby reducing the number of unknowns in the system.

The parameters of the reflected signal are estimated using an Extended Kalman Filter (Gelb, 1979; Brown and Hwang, 1992). Multiple antennas are placed close together to ensure correlated multipath signals. Generally at least five antennas would be used and a typical layout would be (for the six antenna case) as shown in Figure 3.

One of the antennas (normally the center one) would be defined as the reference antenna (A0 in this case). All the reflected signal parameters and the placement of other antennas are defined with respect to this antenna.

The state vector for the estimator is,

$$\begin{bmatrix} \alpha \\ \gamma_0 \\ \theta_0 \\ \varphi_0 \end{bmatrix} = \begin{bmatrix} \text{Reflection coefficient} \\ \text{Reflected signal phase at the reference antenna} \\ \text{Reflected signal elevation} \\ \text{Reflected signal azimuth} \end{bmatrix}$$

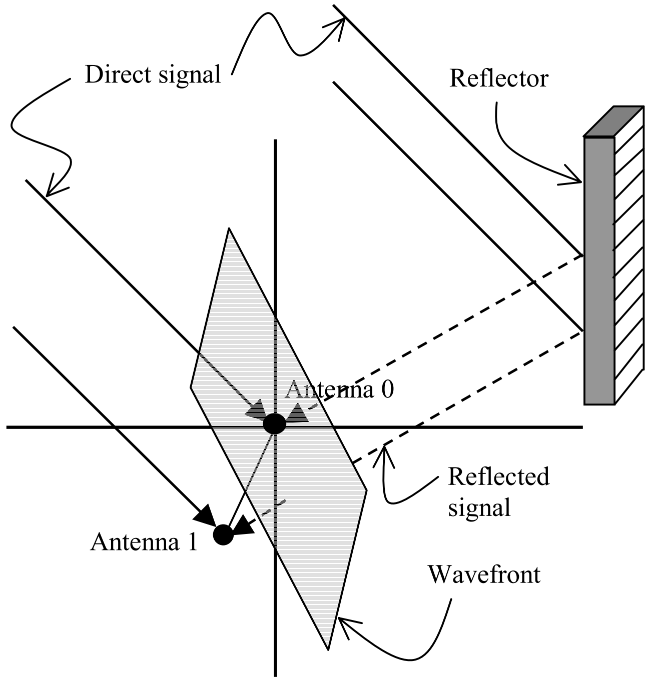

Figure 2: Correlated multipath phase can be related to each other through signal direction and known geometry between the antennas.

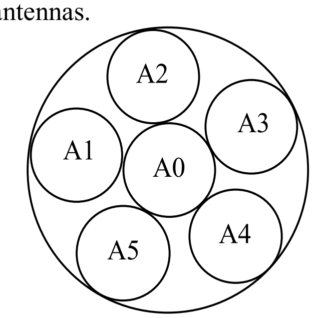

Figure 3: Typical antenna assembly for six antennas.

The modified carrier phase of single differences between antennas are used as measurements in the estimator. The single difference removes most of the errors except receiver clock bias, multipath and carrier phase noise. For example,

$$\Delta\Psi_{0,i} = \Delta\rho_{0,i} + \Delta N_{0,i}\lambda + c\Delta t_{0,i} + \varepsilon_{\varphi 0,i} + \varepsilon_{MP0,i} \quad (4)$$

where,

- $\Delta\Psi$ is the measured carrier phase single difference between antennas 0 and i (m)
- $\Delta\rho$ is the range difference due to spatial separation between antennas
- $c\Delta t$ is the receiver clock bias difference
- ΔN is the integer ambiguity difference
- ε_{φ} is the carrier phase noise difference, and
- ε_{MP} is the carrier phase multipath error difference.

If the receivers are driven by a single external stable clock, the receiver clock bias difference is negligible. In addition, since the phase difference due to the spatial separation of the antennas is known, the range difference then can be eliminated from equation (4) to give,

$$\Delta\Psi'_{0,i} = \Delta N_{0,i}\lambda + \varepsilon_{\varphi 0,i} + \varepsilon_{MP0,i} \quad . \qquad (5)$$

As the multipath induced error is less than a quarter of a cycle, the phase difference due to the relative integer ambiguity can be removed and the residual phase error can be obtained. This residual phase is due to the receiver carrier phase noise and the multipath between the receivers. The relationship between the state variables and the measurements is given in Ray et al. (2000).

The single differenced residual carrier phase error for a particular satellite is input to the multipath mitigating software to adaptively estimate the parameters of the composite multipath signal due to all reflectors affecting the carrier phase. After the parameters are estimated by the filter, it is possible to determine the multipath error in the carrier phase at each of the antennas. The estimated multipath error at each antenna can be differenced and then subtracted from the single differenced phase residual (which was input) to observe a multipath-reduced phase measurement. The signature of the residual phase difference with the multipath error correction (which should be white due to receiver phase noise) can be analyzed to assess the performance of the technique.

By using the above model, it is possible to estimate the carrier phase error due to multipath signals. This method is valid only if the reflected signals are correlated across the antennas, hence the need for closely-spaced antennas on a rigid platform.

TEST DESCRIPTION

In order to test the concept, a special antenna array was assembled whereby a thick aluminum plate was used to rigidly mount six antennas close together, see Figure 4. NovAtel Model 521 antennas were used, as they are small with a diameter of approximately 5.6 cm. The antennas were oriented to the same direction to minimize relative phase center variations.

NovAtel BeeLine receivers were used for data collection (Ford et al., 1997). The BeeLine™ is an 8+8 channel (L1-L1) receiver generally used for the attitude determination. Three BeeLines were used together with six antennas where all receivers were driven by an external rubidium oscillator. Data was collected for several sessions spread over successive days on the roof of the Engineering building at the University of Calgary. Data from October 20, 1998 was used for the following analysis. Only data from four antennas were used in the analysis since the MULTINAV™ software can currently process up to four antennas simultaneously.

Figure 4: Antenna array assembly.

The antenna assembly was placed on a surveyed pillar where there are concrete sidewalls of approximately 3 m in height on the east side and 1 meter in height on the south side. It is expected that these walls, along with the aluminum plate, would cause the most significant multipath signals.

By using Semikin™, a software package developed at the University of Calgary (Cannon, 1993), the position of each of the antennas in the array was determined and their relative geometry established.

The body frame was determined through a one hour static GPS survey, and the resulting body frame coordinates are shown in Table 1. Distances between the GPS antenna pairs ranged from about 8 to 14 cm, and were estimated to an accuracy of less than 1 cm. These were used as constraints in the attitude determination algorithm to eliminate incorrect carrier phase integer ambiguities during the search phase.

Table 1: Antenna body frame coordinates.

Antenna	X (cm)	Y (cm)	Z (cm)
1	0.00	0.00	0.00
2	0.00	12.52	0.00
3	7.76	1.91	0.00
4	-4.83	6.33	2.31

RESULTS

Multipath Mitigation

The single differenced residual described in the previous section contains the carrier phase noise and multipath error. Carrier phase noise is random in nature, while the multipath error is oscillatory where the amplitude depends upon the material and surface structure of the reflector as well as the distance between the reflector and antenna. The frequency of the multipath error is a function of the carrier cycle wavelength and the antenna reflector geometry (Georgiadou and Kleusberg, 1988).

In order to demonstrate the multipath mitigation technique, the single differenced residual phase was computed for satellite 31 (elevation 23°-35°) and is shown in Figure 5. Multipath mitigation was applied to the lower elevation satellites since they are most affected by multipath (Ray, 2000).

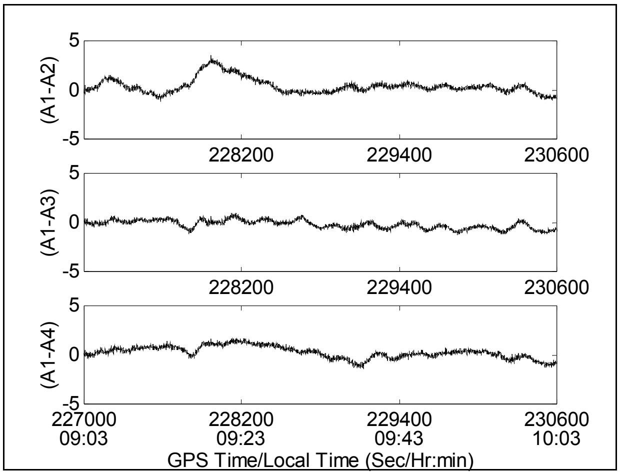

Figure 5: Single differenced residual carrier phase error for SV 31 (Y-axis units in cm; A0-A*n* denotes single difference between antennas 0 and *n*)

The single differenced phase residuals are shown in Figure 5 and contain low to medium frequency oscillations with variable amplitude due to the reflectors in the environment. It also contains high frequency phase noise. Overall, the amplitude reaches up to 3 cm. The multipath-corrupted carrier phase measurement residuals are used as input to the mitigating filter. Figure 6 shows the parameters of the composite reflected signal estimated by the filter for SV 31. The parameters of the virtual reflector vary with time to track the effect of the composite multipath error.

Figure 7 shows the estimated multipath error for SV 31 at each antenna, computed from the estimated parameters of the composite reflected signal. The estimated multipath shows irregular oscillations corresponding to composite multipath, and this demonstrates the capability of the system to estimate a composite reflection effect, rather than a single reflector.

Since the goal is to estimate attitude angles, which uses double differences as the observables to compute the inter-antenna vectors, the double differenced multipath was also computed. This is shown in Figure 8 for one of the antenna baselines.

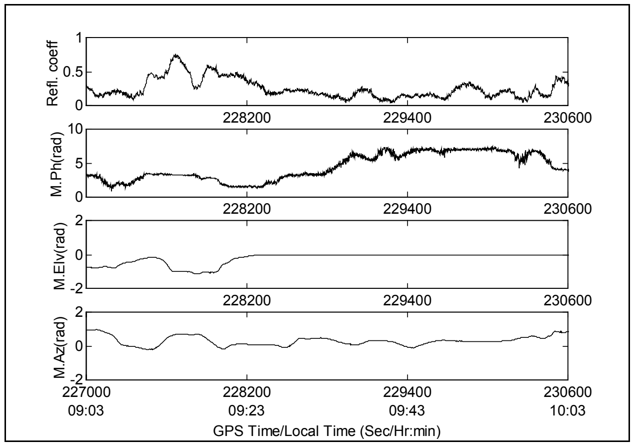

Figure 6: Estimated composite reflected signal parameters for SV 31.

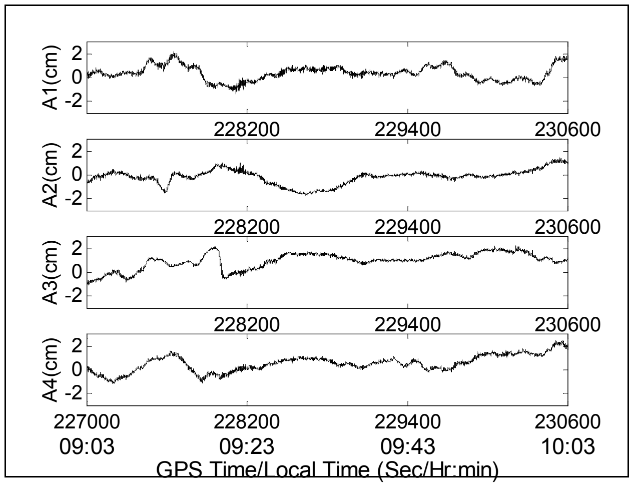

Figure 7: Estimated carrier phase multipath error for SV 31 for each antenna (Y-axis in cm).

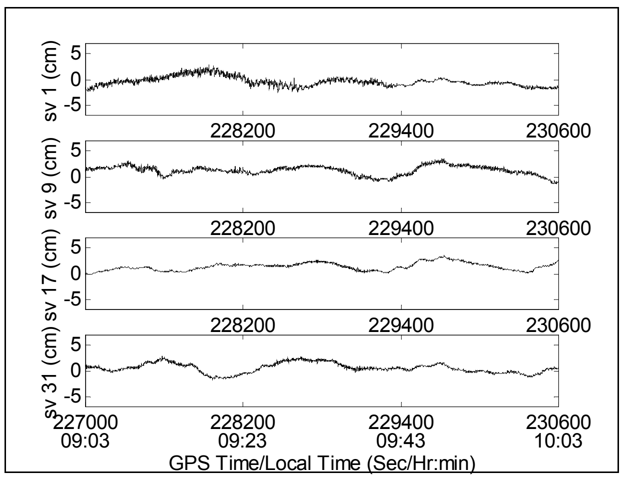

Figure 8: Estimated double difference carrier phase multipath for the Antenna 1-2 baseline.

In order to achieve improved attitude accuracies through multipath mitigation, the residual errors from the raw double difference carrier phase measurements from each antenna pair should be correlated with the values shown on Figure 8. In order to determine this, Figure 9 gives the estimated double difference residuals using raw carrier phase measurements and fixed antenna coordinates. The coordinates were fixed to force all the errors into the residuals.

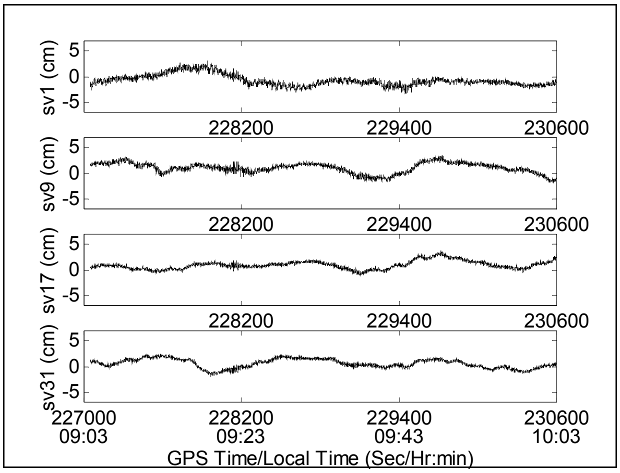

Figure 9: Double difference measurement residuals using raw carrier phase data and constrained coordinates for the Antenna 1-2 baseline.

The correlation between the double difference multipath (Figure 8) and the double difference residuals from the raw measurements (Figure 9) ranged between 73 and 99% which shows the effectives of the method. Indeed, this can be seen in Figure 10 where the double difference measurement residuals were re-computed after the multipath corrections were applied. In this case, the residuals are more random in nature and the phase error due to multipath is nearly eliminated.

This method was applied to the other satellites and baselines during the data collection period and the improvement of residuals is observed in most cases. Table 2 gives an overview of statistics before and after multipath mitigation.

The RMS values of the double differenced multipath-corrected measured phase differences are significantly lower than the values before correction. All the RMS values have been reduced to less than 1 cm. On average, there was a 49% improvement, which clearly demonstrates the effectiveness of this method to mitigate carrier phase multipath in this environment. The percentage improvement in the RMS value is lower than the correlation between the double difference multipath and the double difference residuals from the raw measurements (which ranged between 73 and 99%), as the carrier phase measurement noise is comparable with the multipath error and remains the same before and after multipath mitigation. If the carrier phase noise is removed before computing the RMS error, the percentage of improvement will be much higher.

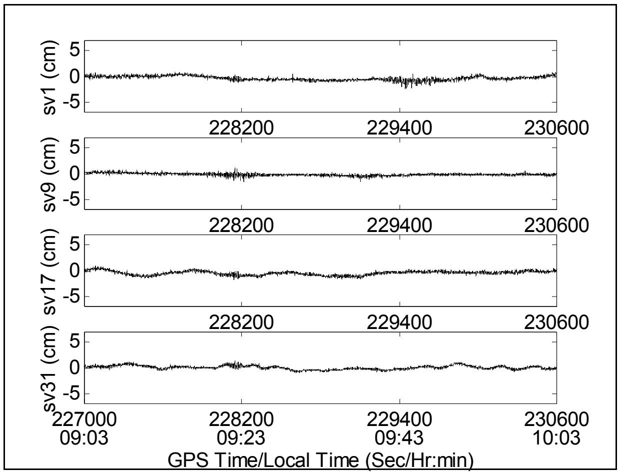

Figure 10: Double difference measurement residuals using corrected carrier phase data and constrained coordinates for the Antenna 1-2 baseline.

Table 2: RMS of double difference measurement residuals before and after corrections using constrained coordinates.

SV	Baseline	Before Correction (cm)	After Correction (cm)	Improvement %
1	1-2	1.34	0.56	58.2
	1-3	2.08	0.33	84.1
	1-4	1.95	0.60	69.2
9	1-2	1.43	0.31	78.3
	1-3	1.38	0.36	73.9
	1-4	0.96	0.70	27.1
17	1-2	1.23	0.63	48.8
	1-3	0.82	0.38	53.7
	1-4	0.75	0.73	2.7
31	1-2	1.05	0.37	64.8
	1-3	0.65	0.76	-15.0
	1-4	0.87	0.51	41.4

Attitude Results

The raw and multipath-corrected carrier phase data was processed using MULTINAV™ and the results were compared for the two cases. Since there was no independent attitude reference, the quality of the results were compared through the consistency as represented by the standard deviation of the angle components.

Figure 11 gives the attitude angles for the raw carrier phase case. The figures clearly show the presence of systematic effects, and over these baseline lengths, it is from multipath. The standard deviation values of the roll, pitch and heading are 5.93, 6.29 and 2.66 degrees, respectively. Given the short distances between the antennas, these achievable accuracies are reasonable.

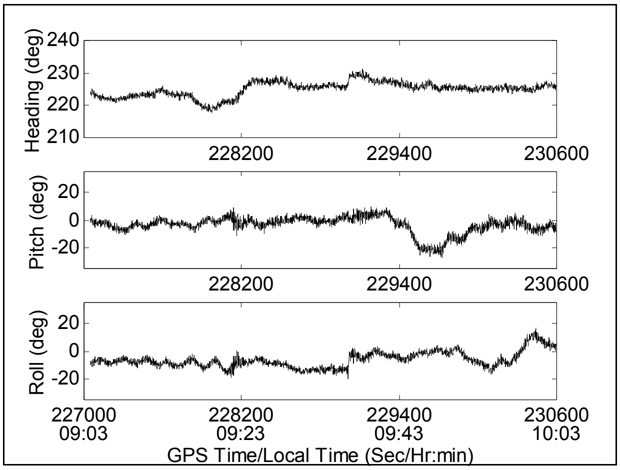

Figure 11: Estimated GPS attitude angles using raw carrier phase data (no multipath corrections).

Figure 12 gives the revised attitude angles after the multipath correction has been applied. The large systematic effects have been reduced significantly, giving better performance overall. The standard deviation values have also been reduced significantly to 4.13, 1.97 and 1.46 degrees, for roll, pitch and heading, respectively. Due to the very short baselines used in this case, consistencies at the several degree level can be expected (Lachapelle et al., 1997).

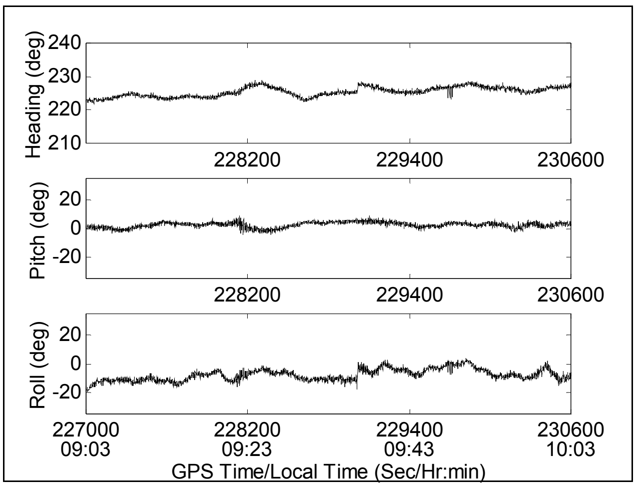

Figure 12: Estimated GPS attitude angles using multipath-corrected carrier phase data.

Table 3 summarizes the statistics and gives the level of improvement as 30.3%, 68.7% and 45.1% for roll, pitch and heading, respectively. The maximum excursions were also reduced significantly, and the 3-D improvement was about 47%. Overall, these results are excellent in terms of improved performance.

The mean values of the attitude components differ by up to several degrees for the raw versus corrected cases. This is due to the multipath, which causes a systematic bias over this one hour timespan. Since the antenna separations are so short, multipath effects on the order of a few cm can easily generate attitude errors of up to several degrees.

Table 3: Attitude angle statistics before and after multipath corrections using constrained coordinates.

1 - D		Before Correction (degrees)	After Correction (degrees)	Improve -ment %
Heading	Mean	225.66	225.38	N/A
	Std Dev	2.66	1.46	45.1
	Max Diff	7.72	3.41	55.9
Pitch	Mean	-0.22	2.47	N/A
	Std Dev	6.29	1.97	68.7
	Max Diff	23.66	6.58	72.2
Roll	Mean	-1.54	-7.77	N/A
	Std Dev	5.93	4.13	30.3
	Max Diff	22.91	11.42	50.2
3 – D Std Dev		9.04	4.80	46.9

CONCLUSIONS

The paper gave an overview of GPS attitude determination using up to four antennas which were separated by distances of 8 to 14 cm. The antennas were mounted on a rigid platform and static data was collected over a one hour period for analysis purposes. A technique to mitigate the carrier phase multipath, based on estimation of multipath parameters from relative carrier phase measurements between antennas, was presented in order to reduce the largest contributor to the attitude error budget.

The technique was shown to be effective in modeling the reflected signal parameters over time from multiple reflectors, and this was demonstrated by computing the double difference phase residuals with and without multipath correction. The RMS values of the residuals were reduced on average by 49%, and in all cases they were less than 1 cm.

Since no independent attitude reference was available, the attitude performance was assessed through the consistency of the results. For the raw carrier phase case, the standard deviation of the attitude angles ranged between 2.7 and 6.3 degrees, while after multipath correction, the RMS values decreased to between 1.5 and 4.1 degrees for a total improvement of about 47%.

The method overall showed effectiveness in mitigating multipath to provide improved performance. It should be noted that this technique is well suited for the case when the environment does not change rapidly over time as the parameter estimation may not be able to model these fluctuations to the required accuracy. It also requires a rigid body platform.

The next steps of this research are to conduct further static tests with various antenna distances and geometries to assess the impact of these variables on the achievable accuracy, and to perform kinematic tests in a benign environment that would simulate a space or airborne case.

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